

# Reliability Analysis of Refining System in Sugar Mill

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**Abstract:** This paper discusses the juice refining system in sugar mill with three states: good, reduced and failed. Two types of failures technical and non-technical are introduced. The failure and repair rates are taken to be constant. Laplace transforms of various probabilities are obtained along with steady state behaviour of the system. Particular case with scheduled maintenance is also given followed by discussion.

**Keywords:** juice refining system, sugar mill.

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## 1. INTRODUCTION

The reliable operation of a system depends upon the capacity, duration and repair facility available to the system. In a sugar mill the juice refining system is an important functionary part. Due to dominant role in production of sugar, this subsystem is required to be maintained in operation. A loss in production is due to inefficient operation of various units of the system. Reliability analysis provides the information about the nature of various units by which the equipment can be maintained in operative conditions.

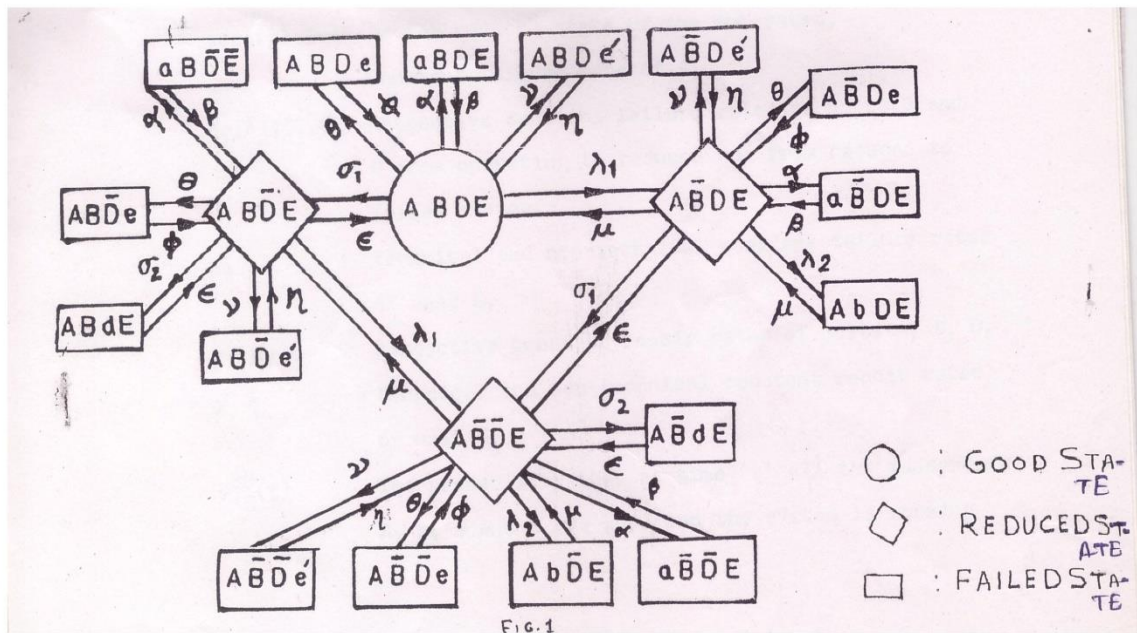
The juice refining system has the operations—filtration, heating, sulphonation, cleaning. The juice from feeding system containing small pieces of baggase and mud first passes through the process of filtration. The filter consists of units in series to ensure the complete removal of baggase from juice. The baggase free juice is first diluted by adding water to increase the fluidity and then sent to the heating unit. The heating unit is similar to heat exchanger to utilize the maximum heat of steam. The steam at higher temperature warms up the juice and gets cooled which is again sent to the boiler. The juice remains in the heater for a definite period of achieve a definite pH value. Heated juice with definite pH value is sent to the sulphonation unit where juice is collected in a tank and sulphur-dioxide is passed through the tank for removing the mud from the juice. The mixture is again passed through heater, because the separation is more clear at higher temperature. The heated mixture is then sent to the clarifier where the cane mud is separated by gravity process. The clarifier vibrates through some mechanism. The heated mixture is stored in the vibrating cavity of the clarifier resulting the separation of clear juice at the upper surface from where it is collected in a tank. The unit has filter having small holes which can be blocked with cane mud. An unskilled worker is there to keep a regular watch on the filter for washing it with water. Technical failure in clarifier is repaired by skilled worker.

## 2. THE MODEL

The system is comprised of four subsystems A, B, D and E as follows:

1. The subsystem (A) consists of two units in series called filter, failure of any one causes the complete failure of the system.
2. The heating subsystem (B) consists of 1 units in parallel, failure of units causes less heating of juice and hence reducing the capacity of the plant. The complete failure occurs only when all units fail.
3. The sulphonation subsystem (D) consists of  $j$  units in parallel, failure of units causes the reduction in capacity and hence loss in production. The complete failure occurs only when all units fail.

4. The clarifier subsystem (E) consists of  $k$  units in series, failure of any one causes the complete failure of the system. The transition diagram of the system is given below:



### 3. ASSUMPTION AND NOTATION

1. Failure and repair rates are constant and statistically independent.
2. A repaired unit is as good as new.
3. B and D have different number of units in parallel and can operate in reduced capacity.
4. Each subsystem has separate repair facility. There is no waiting time for repair in the system.
5. Equipment can be repaired on failure or at the time of first service call after a specified period whichever comes first.
6. Non-technical failure occurs in subsystem (E) only.
7. Service includes repair and/or replacement.

The symbols associated with the system are:

Capital letters for good, small letters for failed and bar on capital letters for reduced state of the subsystem.

- $\alpha$  Constant failure rate of A
- $\lambda_1, \lambda_2, \sigma_1, \sigma_2$  Respective constant failure rates of units B and D from operating to reduced and from reduced to failed states,
- $\theta, \nu$  Technical and non-technical constant failure rates of unit E,
- $\beta, \mu, \varepsilon$  Respective constant repair rates of units A, B, D,
- $\phi, \eta$  Technical and non-technical constant repair rates of unit E,
- $P_{AE}^{BD}(t)$  The probability that at time 't' all the subsystems A, B, D and E are good and the system is working in full capacity. Replacement of A, B, D, E by a, b, d, e and  $\bar{A}, \bar{B}, \bar{D}, \bar{E}$  is for failure and reduced state probabilities.

#### 4. ANALYSIS OF THE SYSTEM

The differential equations associated with the transition diagram are:

$$\begin{aligned} & \left( \frac{d}{dt} + \lambda_1 + \sigma_1 + \theta + \alpha + \nu \right) P_{AE}^{BD}(t) \\ & = \varepsilon P_{AE}^{B\bar{D}}(t) + \mu P_{AE}^{\bar{B}D}(t) + \phi P_{Ae}^{BD}(t) + \beta P_{aE}^{BD}(t) + \eta P_{aE}^{BD}(t) \end{aligned} \quad (1)$$

$$\begin{aligned} & \left( \frac{d}{dt} + \varepsilon + \lambda_1 + \nu + \sigma_2 + \theta + \alpha \right) P_{AE}^{B\bar{D}}(t) \\ & = \sigma_1 P_{AE}^{BD}(t) + \mu P_{AE}^{\bar{B}D}(t) + \varepsilon P_{AE}^{BD}(t) + \phi P_{Ae}^{B\bar{D}}(t) + \beta P_{aE}^{B\bar{D}}(t) + \eta P_{Ae}^{B\bar{D}}(t) \end{aligned} \quad (2)$$

$$\begin{aligned} & \left( \frac{d}{dt} + \mu + \nu + \theta + \lambda_2 + \alpha + \sigma_2 + \varepsilon \right) P_{AE}^{\bar{B}D}(t) \\ & = \lambda_1 P_{AE}^{B\bar{D}}(t) + \varepsilon P_{AE}^{\bar{B}D}(t) + \sigma_1 P_{AE}^{\bar{B}D}(t) + \beta P_{aE}^{\bar{B}D}(t) + \mu P_{AE}^{b\bar{D}}(t) + \phi P_{Ae}^{\bar{B}D}(t) + \eta P_{Ae}^{\bar{B}D}(t) \end{aligned} \quad (3)$$

$$\begin{aligned} & \left( \frac{d}{dt} + \mu + \sigma_1 + \lambda_2 + \alpha + \theta + \nu \right) P_{AE}^{\bar{B}D}(t) \\ & = \mu P_{AE}^{bD}(t) + \lambda_1 P_{AE}^{BD}(t) + \beta P_{aE}^{\bar{B}D}(t) + \phi P_{Ae}^{\bar{B}D}(t) + \eta P_{Ae}^{\bar{B}D}(t) + \varepsilon P_{AE}^{\bar{B}D}(t) \end{aligned} \quad (4)$$

$$\left( \frac{d}{dt} + \varepsilon \right) P_{AE}^{Bd}(t) = \sigma_2 P_{AE}^{B\bar{D}}(t) \quad (5)$$

$$\left( \frac{d}{dt} + \phi \right) P_{Ae}^{B\bar{D}}(t) = \theta P_{AE}^{B\bar{D}}(t) \quad (6)$$

$$\left( \frac{d}{dt} + \beta \right) P_{aE}^{B\bar{D}}(t) = \alpha P_{AE}^{B\bar{D}}(t) \quad (7)$$

$$\left( \frac{d}{dt} + \phi \right) P_{Ae}^{BD}(t) = \theta P_{AE}^{BD}(t) \quad (8)$$

$$\left( \frac{d}{dt} + \beta \right) P_{aE}^{BD}(t) = \alpha P_{AE}^{BD}(t) \quad (9)$$

$$\left( \frac{d}{dt} + \lambda \right) P_{Ae}^{\bar{B}D}(t) = \nu P_{AE}^{\bar{B}D}(t) \quad (10)$$

$$\left( \frac{d}{dt} + \phi \right) P_{Ae}^{\bar{B}D}(t) = \theta P_{AE}^{\bar{B}D}(t) \quad (11)$$

$$\left( \frac{d}{dt} + \beta \right) P_{aE}^{\bar{B}D}(t) = \alpha P_{AE}^{\bar{B}D}(t) \quad (12)$$

$$\left(\frac{d}{dt} + \mu\right) P_{AE}^{bD}(t) = \lambda_2 P_{AE}^{\bar{b}D}(t) \tag{13}$$

$$\left(\frac{d}{dt} + \varepsilon\right) P_{AE}^{\bar{b}d}(t) = \sigma_2 P_{AE}^{\bar{b}\bar{D}}(t) \tag{14}$$

$$\left(\frac{d}{dt} + \eta\right) P_{Ae}^{BD}(t) = \nu P_{Ae}^{BD}(t) \tag{15}$$

$$\left(\frac{d}{dt} + \beta\right) P_{aE}^{\bar{b}\bar{D}}(t) = \alpha P_{aE}^{\bar{b}\bar{D}}(t) \tag{16}$$

$$\left(\frac{d}{dt} + \mu\right) P_{AE}^{b\bar{D}}(t) = \lambda_2 P_{AE}^{\bar{b}\bar{D}}(t) \tag{17}$$

$$\left(\frac{d}{dt} + \phi\right) P_{Ae}^{\bar{b}\bar{D}}(t) = \theta P_{Ae}^{\bar{b}\bar{D}}(t) \tag{18}$$

$$\left(\frac{d}{dt} + \eta\right) P_{Ae}^{\bar{b}\bar{D}}(t) = \nu P_{Ae}^{\bar{b}\bar{D}}(t) \tag{19}$$

$$\left(\frac{d}{dt} + \eta\right) P_{Ae}^{B\bar{D}}(t) = \nu P_{Ae}^{B\bar{D}}(t) \tag{20}$$

with initial conditions  $P_{AE}^{BD}(0) = 1$

= 0 otherwise.

Using Laplace transform technique [1], the probability transforms are given by

$P_{AE}^{BD}(s) = [A(s)]^{-1};$	$P_{AE}^{\bar{b}D}(s) = L[A(s)]^{-1};$
$P_{AE}^{\bar{b}\bar{D}}(s) = M[A(s)]^{-1};$	$P_{AE}^{\bar{b}D}(s) = N[A(s)]^{-1};$
$P_{AE}^{Bd}(s) = \frac{\sigma_2 L}{(s + \varepsilon)} [A(s)]^{-1};$	$P_{Ae}^{\bar{b}\bar{D}}(s) = \frac{\theta L}{(s + \phi)} [A(s)]^{-1};$
$P_{aE}^{\bar{b}\bar{D}}(s) = \frac{\alpha L}{(s + \beta)} [A(s)]^{-1};$	$P_{Ae}^{BD}(s) = \frac{\theta}{(s + \phi)} [A(s)]^{-1};$
$P_{aE}^{BD}(s) = \frac{\alpha}{(s + \beta)} [A(s)]^{-1};$	$P_{Ae}^{\bar{b}\bar{D}}(s) = \frac{\nu N}{(s + \eta)} [A(s)]^{-1};$
$P_{Ae}^{\bar{b}\bar{D}}(s) = \frac{\theta N}{(s + \phi)} [A(s)]^{-1};$	$P_{aE}^{\bar{b}\bar{D}}(s) = \frac{\alpha N}{(s + \beta)} [A(s)]^{-1};$
$P_{AE}^{bD}(s) = \frac{\lambda_2 N}{(s + \mu)} [A(s)]^{-1};$	$P_{AE}^{\bar{b}d}(s) = \frac{\sigma_2 M}{(s + \varepsilon)} [A(s)]^{-1};$

$$P_{Ae}^{BD}(s) = \frac{\nu}{(s+\eta)} [A(s)]^{-1};$$

$$P_{aE}^{\bar{B}\bar{D}}(s) = \frac{\alpha M}{(s+\beta)} [A(s)]^{-1};$$

$$P_{AE}^{b\bar{D}}(s) = \frac{\lambda_2 M}{(s+\mu)} [A(s)]^{-1};$$

$$P_{Ae}^{\bar{B}\bar{D}}(s) = \frac{\theta M}{(s+\phi)} [A(s)]^{-1};$$

$$P_{Ae}^{\bar{B}\bar{D}}(s) = \frac{\nu M}{(s+\eta)} [A(s)]^{-1};$$

$$P_{Ae}^{\bar{B}\bar{D}}(s) = \frac{\nu L}{(s+\eta)} [A(s)]^{-1};$$

where

$$A(s) = s + X_4 - Y_4;$$

$$X_4 = \lambda_1 + \sigma_1 + \theta + \alpha + \nu;$$

$$Y_4 = \varepsilon L + \mu N + \frac{\theta\phi}{(s+\phi)} + \frac{\alpha\beta}{(s+\beta)} + \frac{\nu\eta}{(s+\eta)};$$

$$N = \frac{\lambda_1 + \varepsilon M}{(s + X_1 - Y_1)};$$

$$L = \frac{\sigma_1}{(s + X_3 - Y_3)} \left[ 1 + \frac{\mu\lambda_1}{(s + X_1 - Y_1)(s + X_2 - Y_2)} \right];$$

$$M = \frac{\lambda_1 L}{(s + X_2 - Y_2)} + \frac{\sigma_1 \lambda_1}{(s + X_1 - Y_1)(s + X_2 - Y_2)};$$

$$X_3 = \varepsilon + \lambda_1 + \nu + \sigma_2 + \theta + \alpha;$$

$$X_2 = \mu + \nu + \theta + \lambda_2 + \alpha + \sigma_2 + \varepsilon;$$

$$X_1 = \mu + \sigma_1 + \lambda_2 + \alpha + \theta + \nu;$$

$$Y_3 = \frac{\mu\lambda_1}{(s + X_2 - Y_2)} + \frac{\varepsilon\sigma_2}{(s + \varepsilon)} + \frac{\theta\phi}{(s + \phi)} + \frac{\alpha\beta}{(s + \beta)} + \frac{\nu\eta}{(s + \eta)};$$

$$Y_2 = \frac{\varepsilon\sigma_2}{(s + \varepsilon)} + \frac{\varepsilon\sigma_1}{(s + X_1 - Y_1)} + \frac{\alpha\beta}{(s + \beta)} + \frac{\theta\phi}{(s + \phi)} + \frac{\nu\eta}{(s + \eta)} + \frac{\mu\lambda_2}{(s + \mu)};$$

$$Y_1 = \frac{\mu\lambda_2}{(s + \mu)} + \frac{\alpha\beta}{(s + \beta)} + \frac{\theta\phi}{(s + \phi)} + \frac{\nu\eta}{(s + \eta)}.$$

Probability that the system is operating in full capacity:

$$P_G(s) = P_{AE}^{BD}(s) = [s + X_4 - Y_4]^{-1}$$

Probability that the system is operating in reduced capacity:

$$\begin{aligned} P_R(s) &= P_{AE}^{b\bar{D}}(s) + P_{AE}^{\bar{B}\bar{D}}(s) + P_{AE}^{\bar{B}\bar{D}}(s) \\ &= (L + M + N)(s + X_4 - Y_4)^{-1} \end{aligned}$$

The reliability of the system:

$$R(s) = P_G(s) + P_R(s)$$

$$(l + L + M + N)(s + X_4 - Y_4)^{-1}$$

In steady state as  $t \rightarrow \infty$ ,  $\frac{d}{dt} \rightarrow 0$ , setting  $\frac{d}{dt} = 0$  in equation (1) to (20), we get steady state availability (using normalizing conditions)

$$A(\infty) = \frac{\left[ 1 + \frac{\sigma_1}{\varepsilon} + \frac{\lambda_1}{\mu} + \frac{\lambda_1 \sigma_1}{\mu \varepsilon} \right]}{\left[ \frac{\lambda_1 \lambda_2}{\mu^2} \left( 1 + \frac{\sigma_1}{\varepsilon} \right) + \frac{\sigma_1 \sigma_2}{\varepsilon^2} \left( 1 + \frac{\lambda_1}{\mu} \right) + \left( 1 + \frac{\sigma_1}{\varepsilon} \right) \left( 1 + \frac{\lambda_1}{\mu} \right) \left( 1 + \frac{\theta}{\phi} + \frac{\alpha}{\beta} + \frac{\nu}{\eta} \right) \right]}$$

## 5. THE PARTICULAR CASE

In sugar industry, to overcome the difficulty of total failure and to make the whole system more reliable, the management provides scheduled maintenance at times subject to the existing condition, e.g., capacity, cost, production, raw material etc. The whole plant is repaired after a regular period i.e., shut down period and the plant recovered as new after scheduled maintenance.

The governing equations (1) to (20) reduce to

$$\left( \frac{d}{dt} + \lambda_1 + \sigma_1 \right) P_{AE}^{BD}(t) = u P_{AE}^{\bar{B}D}(t) + \varepsilon P_{AE}^{B\bar{D}}(t) \quad (21)$$

$$\left( \frac{d}{dt} + \mu \right) P_{AE}^{\bar{B}D}(t) = \lambda_1 P_{AE}^{BE}(t) \quad (22)$$

$$\left( \frac{d}{dt} + \varepsilon + \mu \right) P_{AE}^{B\bar{D}}(t) = \sigma_1 P_{AE}^{\bar{B}D}(t) + \lambda_1 P_{AE}^{B\bar{D}}(t) \quad (23)$$

$$\left( \frac{d}{dt} + \varepsilon + \lambda_1 \right) P_{AE}^{B\bar{D}}(t) = \sigma_1 P_{AE}^{BD}(t) + \mu P_{AE}^{\bar{B}D}(t) \quad (24)$$

with initial conditions  $P_{AE}^{BD}(0) = 1$

= 0 otherwise.

The probability transform are given by

$$P_{AE}^{BD}(s) = \frac{(s + \varepsilon)(s + a)(s + \mu)(s + \varepsilon + \lambda_1)}{A}$$

$$P_{AE}^{\bar{B}D}(s) = \frac{\lambda_1 (s + \varepsilon)(s + a)(s + \varepsilon + \lambda_1)}{A}$$

$$P_{AE}^{B\bar{D}}(s) = \frac{\mu \sigma_1 \lambda_1 (2s + a)}{A}$$

$$P_{AE}^{B\bar{D}}(s) = \frac{\sigma_1 (s + \mu)(s + \varepsilon)(s + a) + \mu \sigma_1 \lambda_1 (2s + a)}{A}$$

where

$$A = s[s^4 a^3 + Z_2 s^2 + Z_3 s + Z_4]$$

$$Z_1 = 2s + \sigma_1 + \lambda_1 + \varepsilon$$

$$Z_2 = a^2 + 2a(\lambda_1 + \sigma_1 + \varepsilon) + \varepsilon(\mu + \lambda_1)$$

$$Z_3 = a(\varepsilon + \lambda_1)^2 + 2a\varepsilon(\lambda_1 + \mu) + \sigma_1 \lambda_1 (\varepsilon + \mu) + \mu\varepsilon(\varepsilon + \sigma_1) + a(a\sigma_1 + \mu\lambda_1)$$

$$Z_4 = (\mu + \lambda_1)[\varepsilon a(\varepsilon + \lambda_1) + \sigma_1(a\varepsilon + \mu\lambda_1)]$$

$$a = \varepsilon + \lambda_1 + \mu$$

Reliability of the system after scheduled maintenance is

$$R_{SM}(s) = P_{AE}^{BD}(s) + P_{AE}^{\bar{B}\bar{D}}(s) + P_{AE}^{B\bar{D}}(s) + P_{AE}^{\bar{B}D}(s)$$

$$= \frac{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{s(s^4 + Z_1 s^3 + Z_2 s^2 + Z_3 s + Z_4)} \quad (25)$$

where

$$a_0 = (\mu + \lambda_1)[\varepsilon a(\varepsilon + \lambda_1) + \sigma_1(a\varepsilon + \mu\lambda_1) + \lambda_1 \sigma_1(\mu\varepsilon + \mu a - a\varepsilon)]$$

$$a_1 = a(\varepsilon + \lambda_1)^2 + 2a\varepsilon(\mu + \lambda_1) + \sigma_1 \lambda_1 (\mu + \varepsilon) + \mu\varepsilon(\varepsilon + \sigma_1)$$

$$+ a(a\sigma_1 + \mu\lambda_1) + \lambda_1 \sigma_1 (2\mu - 2\varepsilon - \lambda_1)$$

$$a_2 = a^2 + 2a(\lambda_1 + \sigma_1 + \varepsilon) + \varepsilon(\mu + \lambda_1) + \lambda_1(\mu - \sigma_1 - \varepsilon)$$

$$a_3 = 3(\varepsilon + \lambda_1) + 2\mu + \sigma_1$$

On inversion equation (25) gives reliability function  $R_{SM}(t)$ .

If service facility is always available and it remains busy for time 't' during the interval (0, t]. Let  $C_1$  and  $C_2$  be the revenue per unit time and ordinary service cost per unit time respectively and  $C_3, C_4$  be the respective cost for skilled worker and scheduled maintenance.

The profit function for interval (0, t] is given by

$$H(t) = C_1 \int_0^t R_{SM}(t) dt - C_2 t - (C_3 + C_4)$$

which may be optimised using known methods.

The steady state availability subject to scheduled maintenance is given as

$$A_{SM}(\infty) = \lim_{s \rightarrow 0} s R_{SM}(s) = \frac{a_0}{Z_4}$$

## 6. BEHAVIOUR ANALYSIS

**Table 1: Effect of failure rate of filter, heating plant and clarifier**

Taking  $\beta = 0.4, \mu = 0.1, \varepsilon = 0.2, \phi = 0.15, \eta = 0.4, \nu = 0.02,$

$$\sigma_1 = \sigma_2 = \sigma, \lambda_1 = \lambda_2 = \lambda, \sigma = 0.015$$

$\alpha$	$\lambda$	Availability					$A_{SM}(\infty)$
		$\theta = 0.0$	$\theta = 0.025$	$\theta = 0.05$	$\theta = 0.075$	$\theta = 0.1$	
0.0	0.0	0.97065	0.78111	0.65350	0.56172	0.49255	1.0
	0.01	0.96216	0.77560	0.64964	0.55887	0.49036	0.9998
	0.02	0.94023	0.76129	0.63956	0.55140	0.48460	0.9997
0.005	0.0	0.95902	0.77355	0.64820	0.55781	0.48954	0.9990
	0.01	0.95073	0.76815	0.6440	0.55499	0.48737	0.9982
	0.02	0.92931	0.75411	0.63449	0.54763	0.48168	0.9868
0.01	0.0	0.94765	0.76615	0.64299	0.55395	0.48656	0.9985
	0.01	0.93956	0.76085	0.63925	0.55117	0.48442	0.9980
	0.02	0.91864	0.74707	0.62950	0.54390	0.47880	0.9962

**Table 2: Effect of failure rate of sulphonation plant, technical and non-technical failure in clarifier**

Taking  $\alpha = 0.005, \lambda = 0.01, \beta = 0.4, \mu = 0.1, \varepsilon = 0.2,$

$$\phi = 0.15, \eta = 0.4, \nu = 0.02$$

$\alpha$	$\lambda$	Availability				
		$\theta = 0.0$	$\theta = 0.025$	$\theta = 0.05$	$\theta = 0.075$	$\theta = 0.1$
0.0	0.0	0.97887	0.78642	0.65721	0.56446	0.49466
	0.01	0.95548	0.77125	0.64658	0.55661	0.48892
	0.02	0.93319	0.75666	0.63630	0.54897	0.48272
0.005	0.0	0.97877	0.78319	0.65495	0.56280	0.49338
	0.01	0.95073	0.76815	0.64440	0.55499	0.48737
	0.02	0.92866	0.75368	0.63419	0.54740	0.48150
0.01	0.0	0.96047	0.77450	0.64886	0.55830	0.48992
	0.01	0.93795	0.75979	0.63851	0.55061	0.48399
	0.02	0.91646	0.74563	0.62847	0.54314	0.47820

## 7. DISCUSSION

The foregoing analysis furnishes information about the functioning of the system. The tables for various failure reveals the failure rates of different subsystem upon the availability of the system. From Table 1, we observe that a change in  $\theta$  (failure rate of clarifier  $E$ ) changes the value of availability considerably in comparison to a change in  $\alpha$  and/or  $\lambda_1$ . It shows the effect of clarifier ( $E$ ) upon whole system, hence clarifier ( $E$ ) requires more care. Last column of Table 1 gives the value of availability of the system when scheduled maintenance is considered, from tabular values, it is evident that performing scheduled maintenance the system availability increases considerably, that is why the scheduled maintenance is adopted in sugar mills to run the system satisfactorily for a long period.

## REFERENCES

- [1] E.J. Craig, Laplace and Fourier Transforms for Electrical Engineers, Holt, Reinhart and Winston Inc. N.Y., 1964.
- [2] B.S. Dhillon and C. Singh, Engineering Reliability, New Techniques and Applications. John Wiley, New York (1991).
- [3] I.N. Sneddon, Elements of Partial Differential Equations, McGraw Hill, New York (1987).
- [4] E. Balagurusamy, Reliability Engineering, Tata McGraw Hill, New Delhi, India (1994).